

The Financial Planning and Management of Real Estate Developments

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Introduction

The planning and management of large, multiple-use real estate projects is an extremely difficult task. There are complex and uncertain financial, political, and social factors affecting real estate developments. Major concerns lie in the rather long time horizon and large capital investment necessary to convert land into a merchantable product [2, 4]. Besides technical engineering problems, the real estate developer must deal with the volatile demand for real property and with constantly changing costs over the planning and construction periods. He or she is also influenced by constraints imposed by municipal governments that are troubled by the potential social costs of new developments and by the actions of environmentalists who are concerned about the possible physical and societal impacts of a development [1].

The two basic stages in a real estate development are land development and building construction [5].

The first involves site preparation: grading, utility installation, road construction, and, for residential development, the subdivision of acreage into individual lots. The second stage includes the actual construction of buildings on prepared construction sites. An individual developer may undertake both stages of development in a real estate project or may limit his or her involvement to either land development or building construction.

While mathematical programming has been extensively utilized in other areas of financial management (capital budgeting [3], for example), it has until now received little attention in the real estate development field. Yet, linear programming could be helpful to the management of a real estate development. The real estate developer's problem requires the allocation of scarce resources (land, cash, lines of credit, and so on) to various land uses in different time periods so that

the value of the developer's benefit flows is optimized. Specifically, the developer's problem entails: 1) estimation of the expected future cash receipts and expenditures associated with various land uses and financing conditions; 2) land-use allocations and timing that result in a maximization of the net present value of the future cash flows; and 3) satisfaction of the financial, technological, and societal conditions that limit the prerogatives of the developer.

This paper describes a linear programming model that can be utilized in the financial planning and management of real estate developments. The model considers the market and financial constraints affecting a development and provides a development/construction schedule to maximize the profitability of a real estate project. The framework presented in this paper enables the participants in a real estate development (including not only the developer or investor but also the lenders) to better estimate the financing requirements of a project, as well as to improve the budgeting and control of cash disbursements and receipts. It also allows the developer to determine the effects of deviations from the projected cash budget on future periods of the development and on the overall profitability of the project. By first describing the model in general terms and then presenting an actual application of the model, this paper demonstrates how the real estate industry can use a linear programming model to plan, organize, and control the financial aspects of a very complex production process.

Model Formulation

The general form of the real estate development model is presented in Exhibit 1. Nine types of constraints are considered in this model as representative of the most typical financial and market constraints that the developer faces. To simplify the presentation, environmental and zoning constraints are not included in this model.

Objective Function

The objective of the development model is to maximize the total net present value of the developer's equity. The first term in the objective function represents the present value (discounted at the required rate of return) of the net cash inflows resulting from the sale of the units developed in each period. The development/construction variables (DEV_{hi}) may take the form of several types of land uses such as single-family lots, condominiums, apartments, or commercial units. Depending on the anticipated con-

struction and sales schedules, the cash flows from the development of units may occur not only in the initial development period, but also in future periods as the units are sold. Therefore, the net cash flows from the development of a given land use are the sum over the sales periods of the differences between the cash revenues collected and the cash disbursements from development and construction costs.

The cash outflows in the objective function are the present values of the costs in a given period associated with releasing land from the mortgage (R_{hi}), the mortgage (M_i) interest and amortization payments, the development loan (L_i) interest payments, and the equity (E_i) contributions to the project. The common instrument for financing the land acquisition by a developer is a blanket mortgage [5]. Its terms require the releasing of individual parcels from the mortgage as they are sold, at which time a specified portion of the mortgage debt must be repaid to the lender. The land release variables recognize the amount of land that must be released from the mortgage collateral for sales of each land use. The coefficients (c_i) for these variables are the cost per acre to release land from the mortgage in a given period. These coefficients are a function of not only the outstanding land mortgage and the number of acres securing the mortgage in a period, but also any requirements of the lender governing the total repayment of the mortgage at a specified sales level. For example, the lender may require that the mortgage be repaid by the time the developer has sold 85% of the land securing the mortgage. Since the outstanding balance of the mortgage and the number of acres securing the mortgage decrease over time, the release coefficients in a period are endogenous to the model and are related to the previous period's mortgage balance and release payments. The release cost per acre is thus a non-linear function in the model. Consequently, for linear programming purposes, the release costs must be approximated by a fixed coefficient over a specified time period.

Constraints

While the total land use constraint (1) recognizes the fact that the real estate development is limited by a finite number of acres, the land release constraints (2) determine the amount of land that must be released from the land mortgage in a given period due to the sale of a specific development type in that period. For instance, for each single-family lot sold, sufficient land must be released to account for the size of the lot and

Exhibit 1. General Model

Objective Function

$$\text{Maximize TNPV} = \sum_{h=1}^H \sum_{i=1}^I \sum_{j=1}^{i+n} f_j \text{NCF}_{hji} \text{DEV}_{hi} - \sum_{h=1}^H \sum_{i=1}^I f_i c_i R_{hi} - \sum_{i=1}^I f_i m_i M_i - \sum_{i=1}^I f_i p_i (l/s)_i M_i - \sum_{i=1}^I f_i w_i L_i - \sum_{i=1}^I f_i E_i.$$

Constraints

Total land use constraint:

$$\sum_{h=1}^H \sum_{i=1}^I R_{hi} \leq A. \quad (1)$$

Land release constraints (for $h = 1, \dots, H$ and $i = 1, \dots, I$):

$$\begin{aligned} a_{hi} \text{DEV}_{hi} &= R_{hi} \\ \text{or (in LP form)} \\ a_{hi} \text{DEV}_{hi} - R_{hi} &= 0. \end{aligned} \quad (2)$$

Development loan constraints (for $i = 1, \dots, I$):

$$L_i + E_i + \text{CE}_{i-1} \geq 0. \quad (3)$$

Compensating balance constraints (for $i = 1, \dots, I$):

$$\begin{aligned} L_i + E_i + \text{CE}_{i-1} &\geq uL_i \\ \text{or} \\ (1-u)L_i + E_i + \text{CE}_{i-1} &\geq 0. \end{aligned} \quad (4)$$

Mortgage balance constraints (for $i = 1, \dots, I$):

$$\begin{aligned} M_{i-1} - p_{i-1}(l/s)_{i-1} M_{i-1} - \sum_{h=1}^H c_{i-1} R_{hi-1} &= M_i \\ \text{or} \\ [1-p_{i-1}(l/s)_{i-1}] M_{i-1} - \sum_{h=1}^H c_{i-1} R_{hi-1} - M_i &= 0 \end{aligned} \quad (5)$$

Cash budget constraints (for $i = 1, \dots, I$):

$$\begin{aligned} \text{CE}_{i-1} + E_i + L_i + \sum_{h=1}^H \sum_{k=i-n}^i \text{NCF}_{hik} \text{DEV}_{hk} - \\ \sum_{h=1}^H c_i R_{hi} - w_i L_i - m_i M_i - L_i - p_i(l/s)_i M_i &= \text{CE}_i \\ \text{or} \\ \text{CE}_{i-1} + E_i + \sum_{h=1}^H \sum_{k=i-n}^i \text{NCF}_{hik} \text{DEV}_{hk} - \\ \sum_{h=1}^H c_i R_{hi} - w_i L_i - [m_i + p_i(l/s)_i] M_i - \text{CE}_i &= 0. \end{aligned} \quad (6)$$

Total development loan constraint:

$$\sum_{i=1}^I L_i \leq \text{TL}. \quad (7)$$

Total equity contribution constraint:

$$\sum_{i=1}^I E_i \leq \text{TE}. \quad (8)$$

Market constraints (for $h = 1, \dots, H$):

$$\sum_{i=1}^I \text{DEV}_{hi} \leq \text{MKT}_h. \quad (9)$$

Symbol Definitions (* = Variables with User-Specified Values)

TNPV	= total net present value of the developer on a before-tax basis.
NCF_{hji}	= net cash flows in period j from development type h started in period i where $j = i, i + 1, \dots, i + n$.*
DEV_{hi}	= number of units of type h on which development/construction was started in period i .
c_i	= required mortgage reduction due to the release of one acre of land from the mortgage.*
R_{hi}	= number of acres that must be released for sales of type h in period i .
m_i	= cost of the land mortgage in period i ; <i>i.e.</i> , the interest rate on the mortgage.*
M_i	= outstanding balance of the land mortgage in period i .
w_i	= cost of the development loan in period i ; <i>i.e.</i> , the interest rate on the development loan.*
L_i	= outstanding balance of the development loans in period i .
E_i	= amount of equity contributed to the project in period i .
f_i	= the present value factor for period i ; <i>i.e.</i> , $(1 + r)^{-i}$ where r equals the developer's required rate of return.*
H	= number of land uses (development types).*
I	= number of periods in the development's planning horizon.*
n	= number of periods beyond the initial development period necessary to complete the sales of each land use.*
A	= total number of acres in the project.*
a_{hi}	= required amount of released land per unit of type h due to the sale in period i .
CE_i	= cash balance at end of period i .
u	= percentage compensating balance requirement.*
p_i	= percentage of annual mortgage amortization payment that is due in period i .*
$(1/s)_i$	= sinking fund factor in period i .*
NCF_{hik}	= net cash flows in period i from development type h started in period k where $k = i - n, i - n + 1, \dots, i$.*
TL	= total development loans for project.*
TE	= total available equity for project.*
MKT_h	= market's maximum absorption amount for development type h .*

the necessary road area in front of the lot, as well as a proportionate amount of any common open space in the development. There will be a separate land release constraint for each land use and for each period in the development.

The development loan constraint (3) determines the amount of working capital that must be acquired in a period. If the ending cash balance of the previous period is negative, sufficient capital (either debt or equity) will be obtained to meet any cash requirements from the previous period. The model thus assumes that the developer has a one-period credit arrangement with the suppliers of material and labor. Additionally, it is assumed that the development loans are single-period loans which are either repaid from the net cash flows in the period or "rolled-over" into

the next period.

The compensating balance constraint (4) is a common financial requirement imposed by financial institutions, specifying that the developer maintain a certain percentage (u) of the loan balance as a non-interest bearing deposit with the lender. In periods of negative cash balances, the developer will be required to borrow enough additional capital to not only satisfy constraint (3), but to also meet the compensating balance requirement. If the compensating balance requirement exists ($u > 0$), constraint (3) will not be necessary in the application of the model, since Equation (4) specifies a more demanding constraint than Equation (3).

Constraints (5) and (6) are utilized to determine the outstanding mortgage balance at the beginning of a period and the cash balance at the end of a period. The land mortgage balance is a function of the previous mortgage balance minus the amortization and release payments paid to the mortgage lender during the preceding period. To derive the ending cash balance, the cash budget constraint relies on the accounting identity: beginning cash balance plus cash receipts minus cash outlays equals the ending cash balance. Cash receipts include any equity contributions or development loans plus the net cash inflows from the sales of units developed in the specified period and in previous periods (based upon the sales schedule). Cash outlays would be caused by land release payments, interest and amortization payments on the development loans, and interest and amortization payments on the mortgage. Since it is assumed that the development loans are repaid at the end of the respective periods, these loan flows net out in constraint (6) and do not influence the ending cash balance.

In most real estate projects, there are upper limits on the total loans available to finance the development of the land and on the total amount of equity the developer or investor is willing to commit to a project. Constraint (7) specifies the maximum amount of development loans, and constraint (8) recognizes any equity capital limitations. The final constraint included in the model considers the market absorption rates for alternative land uses. In a specific real estate market, there are bounds on the demand for each type of land use in a given time period. The developer must estimate the maximum absorption rate of the market for each development type, and the market constraints (9) allow the developer to specify these limitations in the model.

Application

The general linear programming model described represents the common objectives and constraints encountered in a real estate development. Individual developments, though, have characteristics unique to the particular site, developer, or lender involved in the

project. Actual application of this model may require modification of the objective function or constraints to reflect the unique circumstances of the development under consideration.

We present here an application of the model to a large, multiple-use, real estate development being planned in Oklahoma City, Oklahoma. The development, called Fox Run, contains a total of 480 acres, and it is expected to require ten years to complete. The model is applied to Phase 1 of the development, which incorporates 162 acres with a three-year development horizon. There are three types of development planned for Phase 1: single-family home lots, multi-family housing sites, and a site for a community shopping center. The developer in this application functions solely as a land developer (first stage of development) and will not undertake any building construction.

The 162 acres in Phase 1 are financed with a 15-year, 6% blanket mortgage having an initial balance of \$671,578. The unusually low interest rate on this mortgage results from the purchase-money financing offered by a governmental agency at the time of the original purchase of the land by the developer. One-third of the annual mortgage amortization is due in the third quarter and two-thirds in the fourth quarter of each year. The developer has negotiated a commitment from a commercial bank to provide 90-day development loans with a 12% annual interest rate. One of the loan conditions is that the total of these loans is limited to \$100,000. Another is that a compensating balance of 10% of any loan must be kept on deposit in the commercial bank.

The developer wants to limit his planned equity contribution in the project to no more than \$10,000. While this level of equity investment may seem unrealistically low, given the size of this project, it should be recognized that real estate developers in these types of activities generally attempt to severely limit their equity exposure. One of the developer's primary goals is the establishment of a schedule that allows the net revenue from early sales to finance the development in succeeding periods. Also, given the uncertainty created by the fluctuating demand for real property and unanticipated changes in development costs, the developer tries to limit his *planned* equity investment in order to maintain an equity reserve that he can fall back on in times of financial distress caused by lengthening sales schedules or rising costs.

An analysis of the single-family housing market in northwest Oklahoma City indicates that the maximum absorption rate for the development of this land use is 126 single-family lots in the first year, 90 lots in

the second year, and 69 lots in the third year of the project. The developer has decided that the best marketing strategy for Phase 1 would be to withhold the multi-family and shopping center sites from the real estate market until a majority of the single-family lots are sold and housing construction is started. Therefore, development of the multi-family and shopping center acres will not commence until the second half of the third year of the project.

The development is modeled on a quarterly basis with an anticipated four-quarter sales schedule for the single-family lots and a two-quarter sell-out time for the multi-family and shopping center acres. It is estimated that 16-2/3% of the single-family lots developed in a period will be sold in that quarter, 33-1/3% will be sold in each of the succeeding two quarters, and 16-2/3% in the third quarter after the period of development. For the multi-family and shopping center sites, it is anticipated that one-half of the acres developed in a specific quarter will be sold in that period with the other half being sold in the following quarter. (An appendix showing the objective function and constraints for application of the model to Phase 1 is available from the authors.)

The coefficients for the development units (DEV_{hi}) are derived from the expected cost and revenue schedules for each of the land-use types. The costs recognized in these coefficients include both the "hard" (materials, equipment, and labor) and "soft" (selling and administrative expenses) development costs, but exclude the land and financing costs (which appear in other terms in the objective function). Given the common nature of some of the development costs (e.g., administrative expenses), some costs are allocated to specific development types based on the relative sales price of the respective development units. The present values of the net cash flows in the objective function are calculated with a developer's annual required rate of return of 12%.

Results

Based on the objective function and constraints, a linear programming solution is obtained for Phase 1 of the Fox Run development. From the results of the optimal solution, the total net present value (TNPV) to the developer of Phase 1 is estimated to be \$1,286,719. The projected development schedule required to produce this TNPV is shown in Exhibit 2. It is interesting to note that, given the limiting financial conditions in this application, no new single-family lots are scheduled for development in the second period of the project. Thus, the only activity in this

Exhibit 2. Projected Development Schedule

Period	Single-Family Lots*	Multi-Family Acres	Shopping Center Acres
1	32		
2			
3	36		
4	33		
5	79		
6	11		
7			
8			
9	69		
10			
11		33	18
12			

*Some values rounded to integers.

period would be the continued development and sale of lots begun in the first period. This schedule would not have been anticipated without the linear programming application, but, given the cash flows generated by the development and sale of single-family lots as well as the debt and equity constraints, this is the development schedule that maximizes the developer's return.

The projected financing schedule derived by the linear programming solution is given in Exhibit 3. The optimal development schedule requires development loans of \$62,971 and \$37,029 in periods two and three, respectively, as well as an equity contribution of \$10,000 in the second quarter of the first year of the development. This financing schedule shows that negative ending cash balances are incurred only in the first six months and that by the end of Phase 1 the cash balance has risen to \$1,779,381. This final cash position includes the cost of reducing the land mortgage from \$671,578 in period one to \$12,490 in period fifteen.

In this application both Equations (7) and (8) serve as binding constraints in Phase 1 — all available development loans and equity are totally utilized in the project. The combination of the negative ending cash balances and the exhaustion of the development capital results in the inability to develop approximately 25 single-family lots that presumably could have been sold in the first year (based on the market absorption projections). Also, given this undeveloped land, 10 of the 162 acres in Phase 1 are not released from the land mortgage.

Parametric programming is employed in this application to determine the effect on the project's development schedule and profitability of alternative

Exhibit 3. Projected Financing Schedule

Period	Mortgage Balance	Development Loans	Equity Contributions	Ending Cash Balance
1	671,578			-66,674
2	666,146	62,971	10,000	-33,326
3	644,418	37,029		0
4	606,838			0
5	553,607			0
6	493,791			210,908
7	412,467			542,559
8	342,177			654,904
9	311,286			516,415
10	300,551			616,949
11	264,834			1,500,875
12	125,299			1,779,954
13	12,739			1,779,763
14	12,739			1,779,572
15	12,490			1,779,381

levels of debt and equity capital. If the developer could renegotiate a higher development loan limit with the commercial bank, then he would be able to not only improve the profitability of the project by increasing the number of single-family lots developed in the first year, but also reduce the required equity commitment. For example, if he can borrow \$125,000 instead of \$100,000, the TNPV of Phase 1 increases by \$36,219, the number of single-family lots developed in the first four quarters increases by nine lots, and the necessary equity capital is reduced from \$10,000 to \$4,691. Additionally, while it is perhaps unlikely that the bank would be willing to totally finance the development of the land, at the \$150,000 loan level the developer need make no equity investment, and the TNPV rises to \$1,329,993.

If the developer could not acquire a higher limitation on the development loans than \$100,000, the profitability of Phase 1 could still be improved by adding equity capital beyond the \$10,000 level. If the developer is willing to invest \$12,531 in the project, an additional nine single-family lots could be developed in the first year, increasing the TNPV to \$1,314,830. While additional equity beyond the \$12,531 level would allow for the total development of all the land in Phase 1, the incremental net present value of the flows from the 16 undeveloped lots would be negative, thus reducing the profitability of the project. Finally, at the other extreme, if the developer with the present development loan limit of \$100,000 wanted to invest no equity capital in Phase 1, the total development would be reduced by 67 single-family lots, and the TNPV would drop to \$1,150,563.

Uncertainty

Thus far the application of the linear programming model to the Fox Run development has assumed all values in the objective function and constraints are known with certainty. Yet, there are two major sources of risk for a real estate developer in these types of development. First, the demand for individual units, and therefore the future sales schedule, is uncertain. Second, there is uncertainty in the estimation of the costs of development in future periods. Lengthening sales schedules and higher costs can result in lower net cash flows in the objective function and changes in the financing requirements through constraint (6). The model, however, can assist the developer in recognizing these uncertainties in both the planning and management stages of the development.

As planning tools, range analysis and sensitivity analysis provide valuable information about the effect of fluctuations in the uncertain variables on the development schedule, financing flows, and profitability of a real estate development. Among the output provided in most linear programming algorithms, range analysis shows the change in the coefficients in the objective function required before there would be a change in the development schedule. For instance, in the Fox Run application, the results of the range analysis indicate that development costs would need to increase by more than 18% before it would be optimal to develop less than 32 single-family lots in the first quarter. Sensitivity analysis can be utilized to test the effect on the development schedule and financing requirements resulting from a major change in the demand for units or development costs.

To illustrate, a new linear programming solution is derived assuming a substantial lengthening of the anticipated sales schedule for the single-family lot from four to eight quarters. The new development schedule resulting from the longer sales schedule is shown in Exhibit 4. Under this new solution, the TNPV of the developer is reduced to \$820,879, and 43 fewer lots are developed during the three-year planning horizon. Also, the decrease in the demand for units significantly reduces the ability of the developer to use revenue from early sales to finance the development in succeeding periods, forcing additional equity investments of \$45,752 in the second quarter and \$136,499 in the third quarter.

Finally, the developer can use this model as a

Exhibit 4. New Development Schedule

Period	Single-Family Lots*	Multi-Family Acres	Shopping Center Acres
1	20		
2	39		
3			
4			
5	35		
6	5		
7	43		
8	6		
9	69		
10			
11		33	18
12			

*Some values rounded to integers.

management tool to monitor and control the diverse facets of the project. As the development unfolds, actual values of uncertain variables become known, and the variable values are fixed for that period. The developer can respond to any unanticipated fluctuations by setting the known values in the model and finding a new solution from that point in the project. Thus, with the real estate development problem structured in a linear programming format, the developer has available a technique for assisting in the management of a difficult production process, especially under conditions of uncertainty.

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