Working Paper 76:2

SENSITIVITY ANALYSIS IN REAL ESTATE INVESTMENT

by

George W. Gau*

and

Daniel B. Kohlhepp*

Center for Economic and Management Research

College of Business Administration

University of Oklahoma

Norman, Oklahoma

April 1976

*Assistant Professors of Business Administration, Division of Finance, University of Oklahoma

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INTRODUCTION

Discounted cash flow techniques have become a widely utilized method of analyzing real estate investments. These procedures have generally been applied in a deterministic framework with single-point estimates of the necessary input variables and implicit assumptions concerning the reinvestment rate of the released capital (any positive cash flows). This paper will expand on the existing procedures to explicitly consider the reinvestment rate and the effect that errors in the estimation of the uncertain inputs will have on the rates of return for real estate projects.

Previous applications of sensitivity analysis to real estate investments have been performed on an *ad hoc* basis for a specific real estate project.[1] Recent studies in capital budgeting by Huefner [4, 5] and Joy and Bradley [6] have investigated the sensitivity question in a more general fashion. Instead of just considering specific investment data, sensitivity analysis was applied to the discounted cash flow models themselves to evaluate the behavior of the techniques given changes in the parameters under consideration. Similar to these studies, this paper will provide a general and theoretical demonstration of sensitivity analysis for real estate investments. Whether a real estate investment analyst is using a deterministic or probabilistic model, the results of this study should allow for a more efficient allocation of evaluation resources in the sensitivity analysis of a specific project. The analyst will be able to distinguish which variables and what error levels have the greatest effect on investment return. Thus, the cost/benefit relationships of additional market information and reduced uncertainty will be evident. Additionally, this study should indicate the relative importance of the reinvestment rate assumption or estimation as a determinant of the rate of return.

BASIC MODEL

The discounted cash flow method identified as the internal rate of return (IRR) calculates the value of a real estate project by determining that rate of discount which equates the net cash inflow of an investment to its initial cash outflows.The IRR model often cited in the analysis of real estate investments takes the basic form:

$$E_o = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{R_n}{(1+r)^n} \tag{1}$$

where

E_o = equity invested in the project at origination (t = 0)

CF_t = net cash flow (after financing charges) in period t

R_n = reversion at the end of holding period t = n

r = rate of return on the equity[2]

Specifically, the two major sources of cash inflow of a real estate project consist of the flows resulting from the operation of the investment during the holding period and changes in the net value of the project by the end of the holding period. The cash flows in each period are determined by the net operating income (O_t), the interest payments on the debt (I_t), the mortgage amortization payments (A_t) and the income taxes accruing to the project (T_t). The reversion flow at the end of period t = n is a function of the selling price of the project (S_n), the capital gains tax (GT_n) and the unpaid mortgage balance at the time of the sale (UM_n).

$$E_o = \sum_{t=1}^{n} \frac{O_t - I_t - T_t - A_t}{(1+r)^t} + \frac{S_n - GT_n - UM_n}{(1+r)^n} \tag{2}$$

Wendt and Wong [12] further divide the net cash inflow stream into two components: a flow derived from the productivity of the investment and a flow resulting from the tax shelter benefits.

$$E_o = \sum_{t=1}^{n} \frac{(1-x)O_t - I_t - A_t}{(1+r)^t} + \sum_{t=1}^{n} \frac{x(I_t+D_t)}{(1+r)^t} + \frac{S_n(1-cx) - UM_n}{(1+r)^n} + \frac{cx(B_o+L_o)}{(1+r)^n} \tag{3}$$

where

x = income tax rate

D_t = depreciation allowance in period t

cx = capital gain tax rate

B_o = building cost at $t = 0$

L_o = land value at $t = 0$

The first term in equation (3) represents the net cash flow without tax shelter benefits; the second term is the tax shelter benefits resulting from leverage and depreciation; the third term is the net cash flow reversion at time of sale; and the final term is the tax shelter benefits derived from the favorable treatment of capital gains.

REINVESTMENT RATE

One of the problems with the application of either the internal rate of return or present value framework is the question of the correct specification of the rate of the reinvestment of the cash flows from the real estate investment. The well-known proposition is that the present value approach implicitly assumes the reinvestment of intermediate cash flows at the discount rate, while the IRR approach assumes the reinvestment is at the derived rate of return. According to this proposition the model specified in equation (1) assumes that the CF_t will be reinvested with a return = r. However, as has been noted by Dudley [3], the reinvestment rate problem results not just from the implicit assumptions but rather because there are no assumptions implicit in the techniques. The assumptions about the reinvestment rates are implicit in the decision to employ a technique and **not** make any explicit estimate of this rate.

Except for the recent article by Messner and Findley [7], in which they form a new decision measure—called a "Financial Management Rate of Return"—that includes estimates of reinvestment rates, existing research in real estate investment analysis has not attempted to explicitly consider the reinvestment rate in a DCF model.[3] One method of developing an alternative IRR model that includes a reinvestment rate $\neq$ r is through the initial employment of the capital budgeting decision criterion of net terminal value (NTV).[4] In terms of our basic model, the NTV of the equity will equal:

$$NTV = \sum_{t=1}^{n} \left[CF_t \prod_{\tau=t+1}^{n} (1+i_\tau) \right] + R_n - E_o(1+k)^n \qquad (4)$$

where

k = required rate of return on the equity

i_τ = reinvestment rate in periods t+1 to n.

If i_τ is assumed to be constant over the holding period,

$$NTV = \sum_{t=1}^{n} CF_t(1+i)^{n-t} + R_n - E_o(1+k)^n \tag{5}$$

The project return can be derived from the preceding NTV formulation by finding the r that will equate E_o in NTV terms with CF_t and R_n in NTV terms.

$$E_o(1+r)^n = \sum_{t=1}^{n} CF_t(1+i)^{n-t} + R_n \tag{6}$$

Dividing through by $(1 + r)^n$ to transform equation (6) into present value terms,

$$E_o = \sum_{t=1}^{n} \frac{CF_t(1+i)^{n-t}}{(1+r)^n} + \frac{R_n}{(1+r)^n} \tag{7}$$

Equation (7) thus represents an IRR model that explicitly considers the effect of the reinvestment rate on the rate of return of a real estate investment.

Notice that, if $i = r$, equation (7) reduces to the basic model, equation (1). Substituting r for i in equation (7),

$$E_o = \sum_{t=1}^{n} \frac{CF_t(1+r)^{n-t}}{(1+r)^n} + \frac{R_n}{(1+r)^n} \tag{8}$$

$$E_o = \sum_{t=1}^{n} CF_t(1+r)^{-t} + \frac{R_n}{(1+r)^n} \tag{9}$$

$$E_o = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} + \frac{R_n}{(1+r)^n} \tag{1}$$

ESTIMATION ERRORS

This paper will examine the effect on r of different error rates in the estimation of the input values, including the reinvestment rate. If we assume that the cash flows will decline at a given rate α each period and that the equity value of the property grows at a certain rate β each period, then[5]

$$CF_t = CF_1 (1-\alpha)^{t-1} \tag{10}$$

$$R_n = E_o (1+\beta)^n \tag{11}$$

and equation (7) becomes

$$E_o = \sum_{t=1}^{n} \frac{CF_1 (1-\alpha)^{t-1}(1+i)^{n-t}}{(1+r)^n} + \frac{E_o (1+\beta)^n}{(1+r)^n} \tag{12}$$

Simplifying,

$$E_o = CF_1 \sum_{t=1}^{n} \frac{(1-\alpha)^{t-1} (1+i)^{n-t}}{(1+r)^n} + E_o \frac{(1+\beta)^n}{(1+r)^n} \tag{13}$$

Dividing through by E_o,

$$\frac{CF_1}{E_o} \left[\sum_{t=1}^{n} \frac{(1-\alpha)^{t-1} (1+i)^{n-t}}{(1+r)^n} \right] + \frac{(1+\beta)^n}{(1+r)^n} - 1 = 0 \tag{14}$$

In the preceding model, equation (14), the uncertainty rests in the estimation of CF_1/E_o, α, β, n and i.

1. Cash flows (CF_1 and α)
   The deviations of the estimated values of these inputs from the realized or actual quantities would be caused by unanticipated changes in factors such as occupancy rates and operating expenses.
2. Amount of equity required in the investment (E_o)
   While in many cases this variable can be estimated with a high degree of certainty,

changes in construction costs (inaccurate cost estimates, delays, price changes, etc.) or financial arrangements would result in deviations of E_o.

3. Reversion flow at the end of the holding period (β)

   The primary determinant of R_n is the selling price of the investment in period $t = n$. Among the uncertain variables that would affect the selling price include market demand and supply factors and inflation rates.

4. Length of the holding period (n)

   Under certain conditions (the economic life of the investment is longer than the desired holding period) n may be a predetermined, nonstochastic variable. Yet for any investment there is a degree of uncertainty concerning the length of the time period in which a real estate project will produce a sufficient return.

5. Reinvestment rate (i)

   The estimation of the reinvestment rate requires the anticipation of the expected returns available on alternative investments. The actual i would deviate from its expected value if there are changes in the conditions in the financial markets, interest rates, etc.

The error rates for the input data are defined as the actual (realized) value of the variable minus its estimated (expected) value divided by the estimated value. The error (deviation) rates would be (e = estimated value and a = actual value):

1. Percentage error of cash flow coefficient (D_c)

$$\frac{(CF_1/E_o)^a - (CF_1/E_o)^e}{(CF_1/E_o)^e}$$

2. Percentage error of α (D_α)

$$\frac{\alpha^a - \alpha^e}{\alpha^e}$$

3. Percentage error of β (D_β)

$$\frac{\beta^a - \beta^e}{\beta^e}$$

4. Percentage error of n (D_n)

$$\frac{n^a - n^e}{n^e}$$

5. Percentage error of reinvestment rate (D_i)

$$\frac{i^a - i^e}{i^e}$$

6. Percentage error of rate of return (D_r)

$$\frac{r^a - r^e}{r^e}$$

The effect of these errors of estimation on the rates of return will be positive (actual r based on the actual value of the variables is greater than the estimated r based on the estimated values of these variables, $r^a > r^e$ and $D_r > 0$), negative ($r^a < r^e$ and $D_r < 0$) or neutral ($r^a = r^e$ and $D_r = 0$).

METHODOLOGY

A simulation study was used to examine the sensitivity of the rate of return (r^e) to errors of estimation (D). Equation (14) was programmed and numerous runs were made, beginning with the base case calculation of r^e and then adjusting for estimation errors of various levels. The base case calculations of r^e was made assuming variable values for real estate investments as follows[6]

$$\begin{aligned}
(CF_1/E_0)^e &= .07 \\
\alpha^e &= -.03 \\
\beta^e &= .06 \\
i^e &= .12
\end{aligned}$$

Using these values, the internal rate of return (r^e) was calculated for three holding periods (n = 5, 10, 20). Given the r^e for each holding period, the effect of errors of estimation on the r^e for each holding period was determined by reducing a designated variable by 50 percent and 25 percent and then increasing the variable by 25 percent and 50 percent, holding the other variables constant.[7] The percentage change in r^e, (D_r), was calculated as well as the elasticity of r^e for each change.

The preceding procedure was repeated as the value for each variable was changed. $(CF_1/E_0)^e$ was changed from .07 to .14, and α was changed from a positive growth rate in cash flows ($\alpha = -.03$)[8] to a negative growth rate in cash flow ($\alpha = .03$). The growth rate of the original equity, β, was increased from .06 to .12. These changes were made to consider the effect of errors of estimation in cases of alternative levels of the variables. Finally, the reinvestment rate, i, was changed from .12, which was close to the internal rate of return in the base case, to .06, which was closer to a "safe rate" of reinvestment.

RESULTS

The results of the simulation runs are summarized in a tabular form in Table 1 and also in graphical form in Figures 1 through 6.[9] The effect of the reinvestment rate on the internal rate of return is demonstrated in Figure 6.

In all cases the elasticities were less than one. This indicates that a specified error in any of the estimated variables did not result in a proportional variance in the rate of return; in fact, the highest elasticity observed was .594 (Table 1A).

The runs for the base case indicate that, for the five and ten year holding period, the relative importance of estimation errors in descending order are: (1) the first year equity dividend rate, $(CF_1/E_0)^e$; (2) the growth rate of the original equity, β^e; (3) the reinvestment rate, i^e; (4) the holding period, n^e; and (5) the growth rate of the cash flows, α^e. However, in the 20-year holding period case, the rate of return is most sensitive to errors in i^e followed by errors in the $(CF_1/E_0)^e$ and β^e. Errors in a α^e have a greater impact on r than do errors in n^e. These results are depicted in Figure 1.

The following generalizations are made from the base and alternative case runs:

1. The greater the absolute value of D_c, D_α, D_β, D_i and D_n, the greater the absolute value of D_r.
2. In all cases errors of estimation produce positive elasticities except for D_n and D_α when $\alpha = .03$.

3. The larger the values of $(CF_1/E_0)^e$, β^e, i^e and n^e, the greater the effect of errors of estimation on r^e. A positive α^e has a lesser effect on r^e than a negative α^e of the same absolute value.
4. The longer the holding period, the lesser the effect of D_c and D_β on D_r.
5. The longer the holding period, the greater the effect of D_α and D_i on D_r.

Of particular interest in this study was the effect of the reinvestment rate estimate on the internal rate of return. The results of the study indicate that, as the reinvestment rate increases, the rate of return also increases, e.g., as i^e increases from 6 percent to 12 percent, r^e increases from 11.9 percent to 12.6 percent, and as i^e increases from 12 percent to 18 percent, r^e increases from 12.6 percent to 13.3 percent. Furthermore, the higher the reinvestment rate becomes, the more sensitive the rate of return becomes to errors in the reinvestment rate, e.g., in Table 1D, $E_{r:i} = .149$ when $i^e = .06$, $D_i = -50\%$ and $n^e = 20$ and $E_{r:1} = .341$ when $i^e = .12$, $D_i = -50\%$ and $n^e = 20$.[10] Also, as the holding period increases, the rate of return becomes increasingly sensitive to errors in the reinvestment rate, e.g., $E_{r:i} = .103$ when $i^e = .12$, $D_i = -50\%$ and $n^e = 5$ and $E_{r:i} = .341$ when $i^e = .12$, $D_i = -50\%$ and $n^e = 20$.

The magnitude of the reinvestment rate relative to the internal rate of return affects the importance of the holding period estimate. When the reinvestment rate estimate is very close to the rate of return, the effect of the errors in the holding period is very small, e.g., $E_{r:n} = -.057$ when $r^e = .122$, $i^e = .12$, $n^e = 10$ and $D_n = -40\%$. However, when the reinvestment rate is considerably different from the internal rate of return, the errors in the holding period are relatively more important, e.g., $E_{r:n} = -.182$ when $r^e = .163$, $i^e = .12$, $n^e = 10$ and $D_n = 40\%$.

An interesting result from a survey of the elasticities listed in Table 1E and the slopes of the graphs of D_n in the figures should be noted. In all cases $E_{r:n}$ and the slopes are both negative, meaning that a negative error in the estimation of the holding period ($n^a < n^e$) causes a higher rate of return ($r^a > r^e$) and, conversely, a positive D_n produces a $D_r < 0$. This finding, on the surface, would seem to contradict accepted financial reasoning, i.e., if an investment has positive cash flows but the economic life of the project turns out to be less than the expected holding period, the resulting rate of return should fall below the rate expected with the original holding period. Yet, in this study the rate of return increases to a higher level than originally expected. For example, in Table 1E, with the cash flows growing at the rate of 3 percent and $n^e = 20$ but $n^a = 12$, the rate of return rose from $r^e = 11.8\%$ to $r^a = 12.1\%$. The explanation for this consistent result across the cases rests in the relationship between the level of the reinvestment rate and the level of the rate of return.

When the estimated reinvestment rate is less than the resulting rate of return estimate, positive errors in the holding period estimate cause decreases in the actual rate of return. With two exceptions, the basic and alternative cases considered so far in this study had $i^{e'}s < r^{e'}s$. This relationship means that in these cases the CF_t's are expanding at a rate i^e insufficient to cover the rate of discount of the flows, and with each additional year of the holding period, the resulting r^e will be reduced to a lower level. Therefore, the longer the holding period, the lower the derived rate of return for the investment. In the two runs where $i^e > r^e$ (Figure 3, $n^e = 10$ and $n^e = 20$), the growth rate of the cash flows (α) was a -3 percent, and the combination of the declining cash flows with i^e still resulted in a total expansion of flows at a rate $< r^e$.

When the estimated reinvestment rate (net of α) is greater than the estimated internal rate of return, positive errors in the holding period produce increases in the actual rate of return. In this situation each addition to the holding period brings about a higher r^e since the rate of reinvestment of CF_t is at a level higher than the rate of discount. To demonstrate this type of result, a supplemental case was established and runs were made with $i^e = .18$, $(CF_1/E_0)^e = .07$, $\alpha^e = .03$, $\beta^e = .06$, $n^e = 20$, $r^e = .133$ and positive/negative D_n. These runs are presented in Figure 7. In this case $E_{r:n} > 0$, and the slope of D_n is positive.

CONCLUSION

The results of this study indicate that a real estate investment analyst is well-advised to allocate his resources toward obtaining good initial point estimates. The relative sizes of the elasticities of the uncertain inputs suggest that, for projects with short holding periods, the critical estimates are: the initial required equity, E_0; the first year cash flow, CF_1; and the growth rate of the equity, β. For projects with longer holding periods, the real estate analyst should be especially concerned with the estimation of the reinvestment rate for the released capital. The benefits of reducing the uncertainty surrounding these inputs can be compared to the costs of the reduction. For instance, the costs of reducing D_c from -50 percent to -25 percent by better market information or lease arrangements should be compared to the corresponding benefit of reducing D_r. This type of cost-benefit analysis should be the basis for determining the decision makers' resource allocation in their sensitivity analysis of a real estate project. This study provides information to the analyst concerning the optimum resource allocation.

Since CF_1/E_0, β and i were found to be the most important variables affecting r, an interesting finding of this study is that, in a traditional IRR model (with the assumption that $r = i$), a project with a short holding period should be viewed as having a higher level of risk than a project with a longer holding period. As an example of the evidence supporting this proposition, consider Figure 2 and the value of D_r, given −50 percent errors of the input variables and alternative holding periods. In all these graphs the two significant variables in terms of positive elasticities are CF_1/E_0 and β– ignoring i, since in the traditional model the reinvestment rate is not an uncertain input. At −50 percent values for D_c, D_r falls from −30 percent when $n^e = 5$ to −20 percent when $n^e = 20$. Similarly for D_β, D_r drops from approximately −10 percent to −4 percent. The smaller values of D_r show that any errors in the estimation of these variables have less impact the longer the holding period. On this basis, these results are contrary to the basic premise of the payback period criteria which tends to favor short-lived projects.

Finally, the results of this study clearly demonstrate that the reinvestment rate must be considered a major determinant of a project's rate of return and a project's risk, especially when a long holding period is anticipated. As such, it should be explicitly entered into discounted cash flow models rather than assumed to be equal to the internal rate of return. As shown in this study, decisions concerning the optimum holding period of a project must recognize the effect of the reinvestment rate on this decision. The failure to explicitly include the reinvestment rate as an input variable in the IRR model will result (if there are differences between the levels of i and r) in the derived rate of return being an inaccurate measure of the profitability of a real estate investment.

TABLE 1

A

r^e	Variable	Estimate	Percentage Error	Holding Period	Percentage Change in Return	Elasticity
$r^e_{n=5} = .126$	CF_1/E_o	= .07	–25	5	–11.9	.477
				20	– 8.5	.399
$r^e_{n=20} = .118$			–50	5	–24.6	.492
				20	–18.6	.373
$r^e_{n=5} = .179$	CF_1/E_o	= .14	–25	5	–14.2	.566
				20	–.086	.344
$r^e_{n=20} = .146$			–50	5	–29.7	.594
				20	–19.4	.389

B

r^e	Variable	Estimate	Percentage Error	Holding Period	Percentage Change in Return	Elasticity
$r^e_{n=5} = .126$	β	= .06	–25	5	– 9.4	.376
				20	– 4.2	.17
$r^e_{n=20} = .118$			–50	5	–18.5	.371
				20	– 7.7	.154
$r^e_{n=5} = .174$	β	= .12	–25	5	–14.1	.563
				20	–11.4	.457
$r^e_{n=20} = .148$			–50	5	–27.8	.556
				20	–20.2	.40

C

r^e	Variable	Estimate	Percentage Error	Holding Period	Percentage Change in Return	Elasticity
$r^e_{n=5} = .126$	α	= –.03	–25	5	– .6	.026
				20	–1.5	.061
$r^e_{n=20} = .118$			–50	5	–1.3	.025
				20	–3.0	.059
$r^e_{n=5} = .12$	α	= .03	–25	5	.6	–.023
				20	1.2	–.049
$r^e_{n=20} = .106$			–50	5	1.2	–.023
				20	2.5	–.049

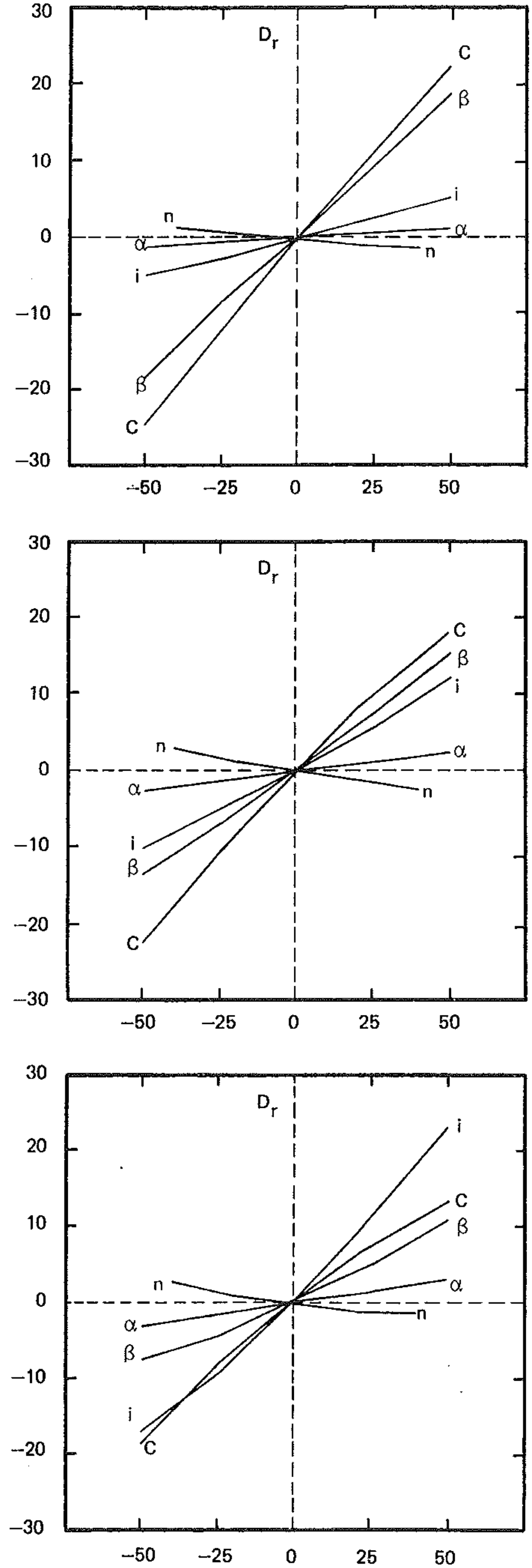
D

r^e	Variable	Estimate	Percentage Error	Holding Period	Percentage Change in Return	Elasticity
$r^e_{n=5} = .126$	i	= .12	–25	5	–2.7	.108
				20	–9.3	.373
$r^e_{n=20} = .118$			–50	5	–5.2	.103
				20	–7.3	.341
$r^e_{n=5} = .119$	i	= .06	–25	5	–1.3	.159
				20	–3.4	.159
$r^e_{n=20} = .098$			–50	5	–2.6	.052
				20	–7.5	.149

E

r^e	Variable	Estimate	Percentage Error	Holding Period	Percentage Change in Return	Elasticity
$r^e = .126$	n	= 5	–20	5	.7	–.028
			–40	5	1.5	–.03
$r^e = .118$	n	= 20	–20	20	1.0	–.05
			–40	20	2.45	–.061

FIGURE 1



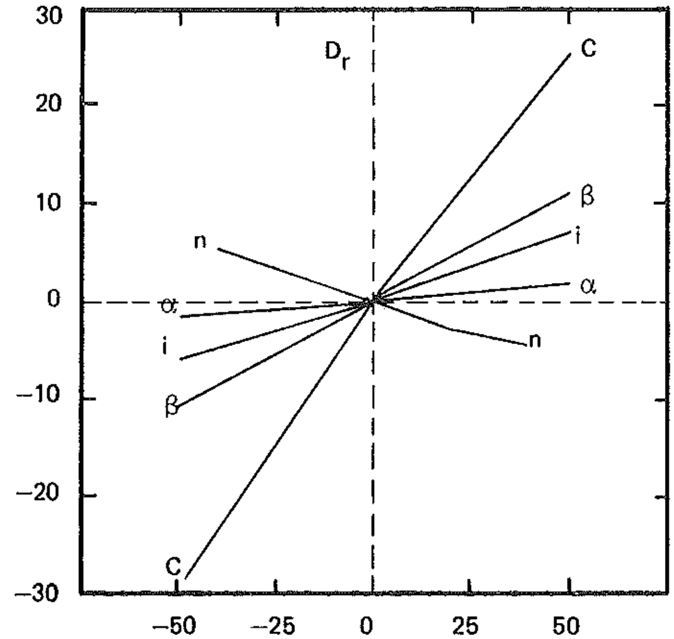
$(CF/E_o)^e$ = .07
α^e = –.03
β^e = .06
i^e = .12
n^e = 5
r^e = .126

$(CF/E_o)^e$ = .07
α^e = –.03
β^e = .06
i^e = .12
n^e = 10
r^e = .122

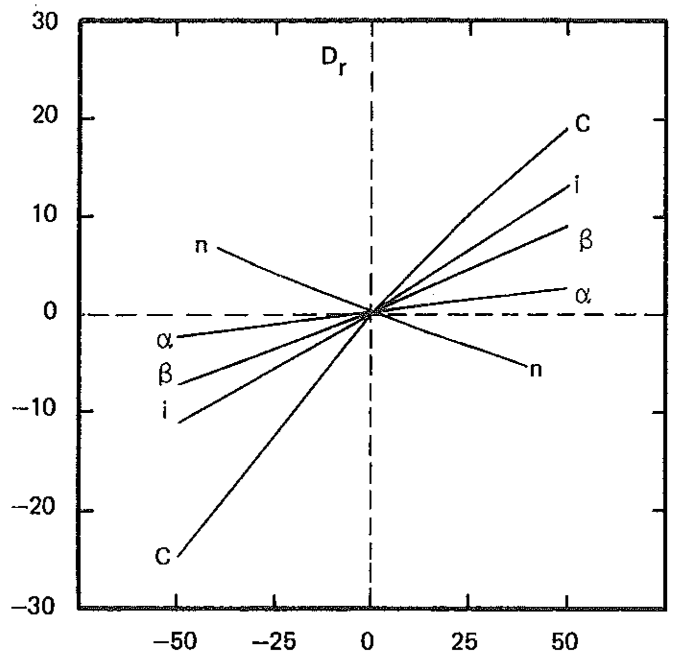
$(CF/E_o)^e$ = .07
α^e = –.03
β^e = .06
i^e = .12
n^e = 20
r^e = .118

FIGURE 2

$^*(CF/E_o)^e = .14$
$\alpha^e = -.03$
$\beta^e = .06$
$i^e = .12$
$n^e = 5$
$r^e = .179$



$(CF/E_o)^e = .14$
$\alpha^e = -.03$
$\beta^e = .06$
$i^e = .12$
$n^e = 10$
$r^e = .163$



$(CF/E_o)^e = .14$
$\alpha^e = -.03$
$\beta^e = .06$
$i^e = .12$
$n^e = 20$
$r^e = .146$

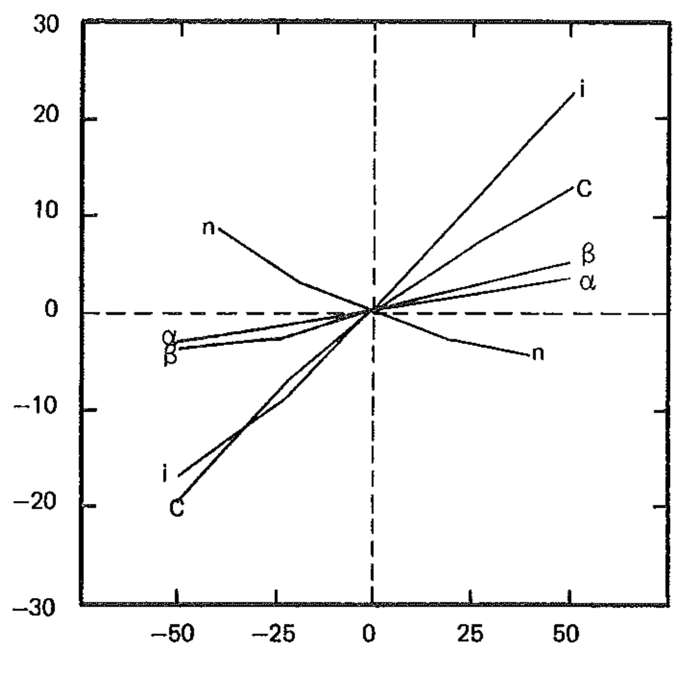
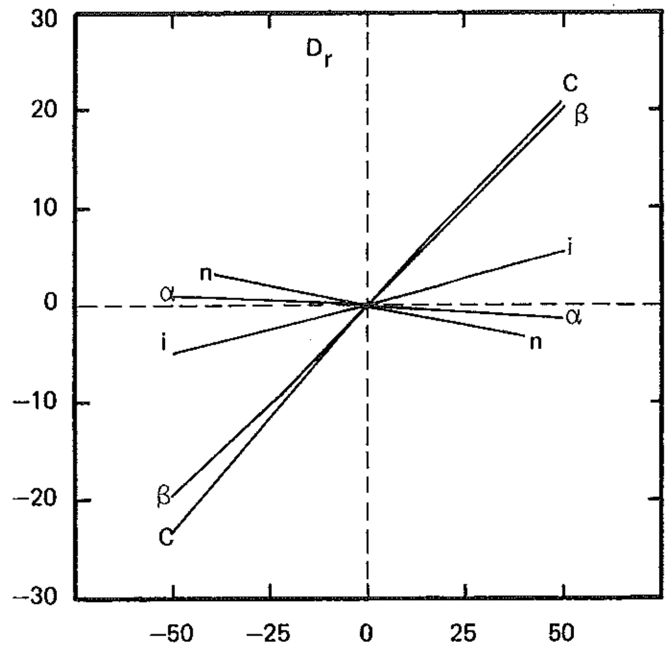
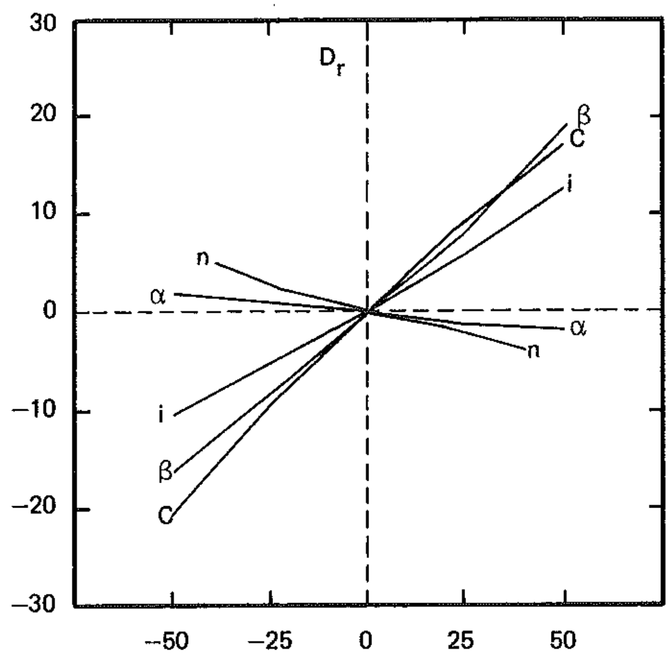


FIGURE 3

$(CF/E_o)^e$ = .07
$^*\alpha^e$ = .03
β^e = .06
i^e = .12
n^e = 5
r^e = .12



$(CF/E_o)^e$ = .07
α^e = .03
β^e = .06
i^e = .12
n^e = 10
r^e = .112



$(CF/E_o)^e$ = .07
α^e = .03
β^e = .06
o^e = .12
n^e = 20
r^e = .106

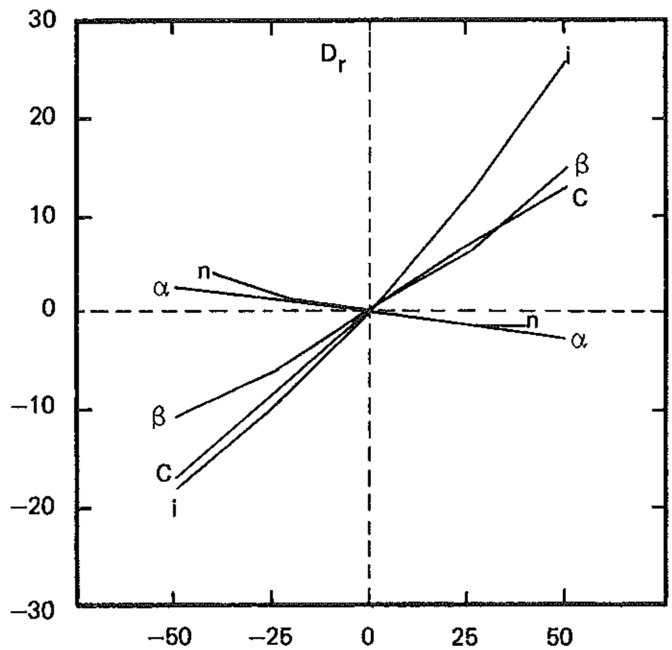
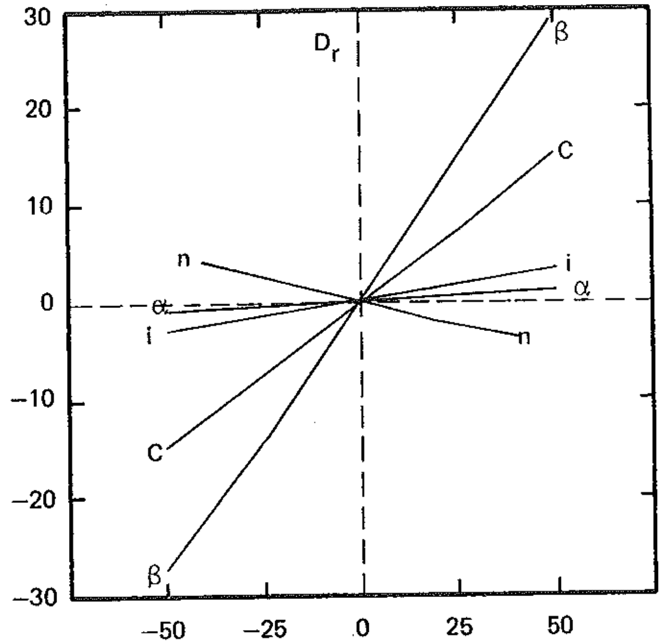
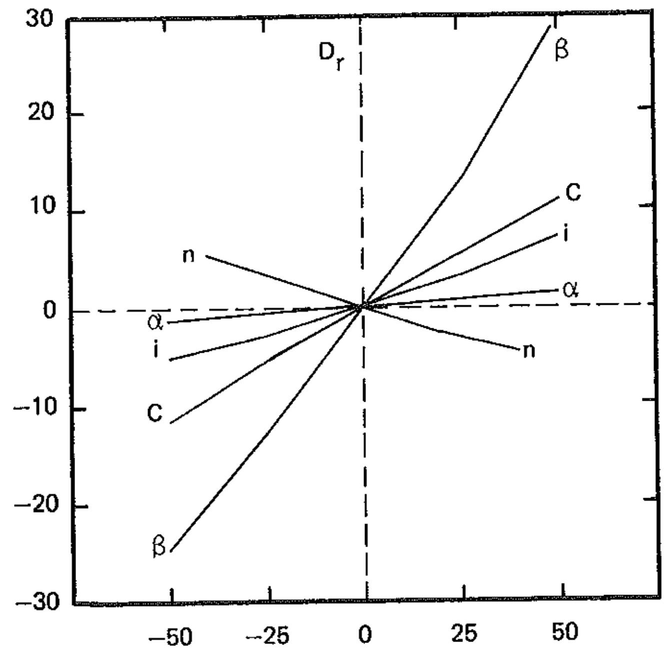


FIGURE 4

$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$^*\beta^e = .12$
$i^e = .12$
$n^e = 5$
$r^e = .174$



$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$\beta^e = .12$
$i^e = .12$
$n^e = 10$
$r^e = .162$



$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$\beta^e = .12$
$i^e = .12$
$n^e = 20$
$r^e = .148$

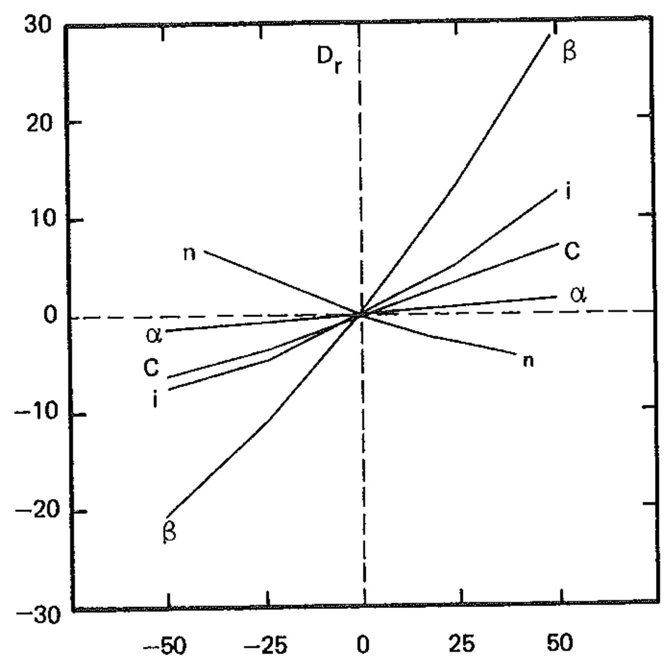


FIGURE 5

$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$\beta^e = .06$
$*i^e = .06$
$n^e = 5$
$r^e = .119$

$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$\beta^e = .06$
$i^e = .06$
$n^e = 10$
$r^e = .11$

$(CF/E_o)^e = .07$
$\alpha^e = -.03$
$\beta^e = .06$
$i^e = .06$
$n^e = 20$
$r^e = .098$

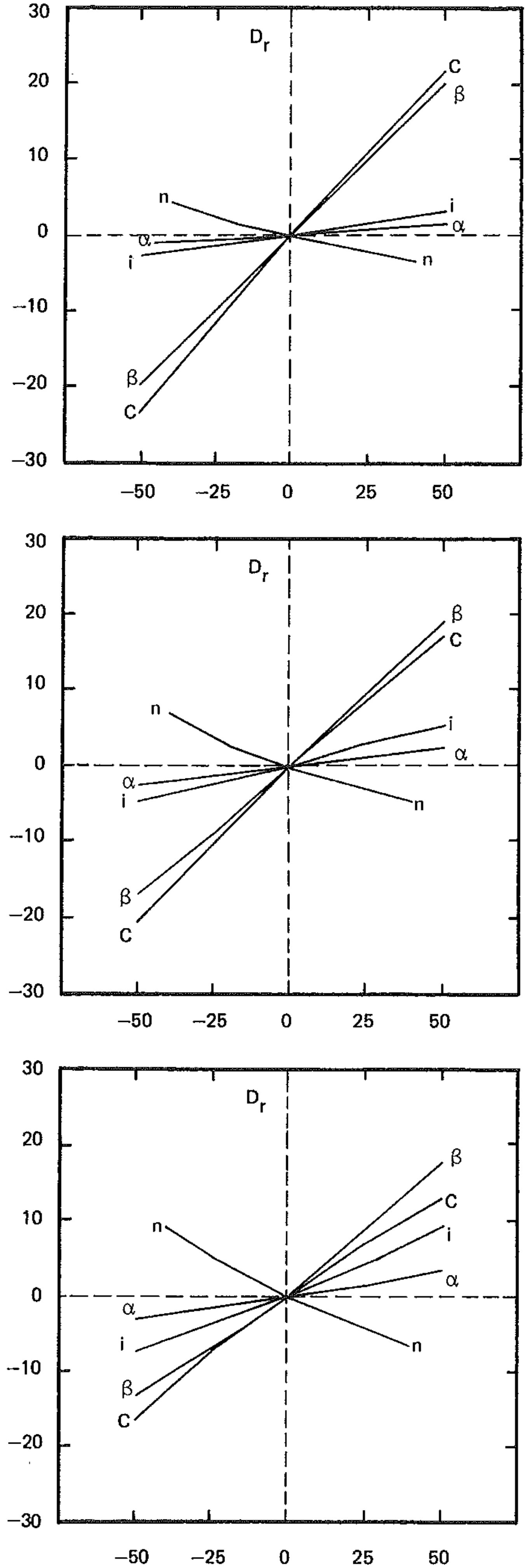


FIGURE 6

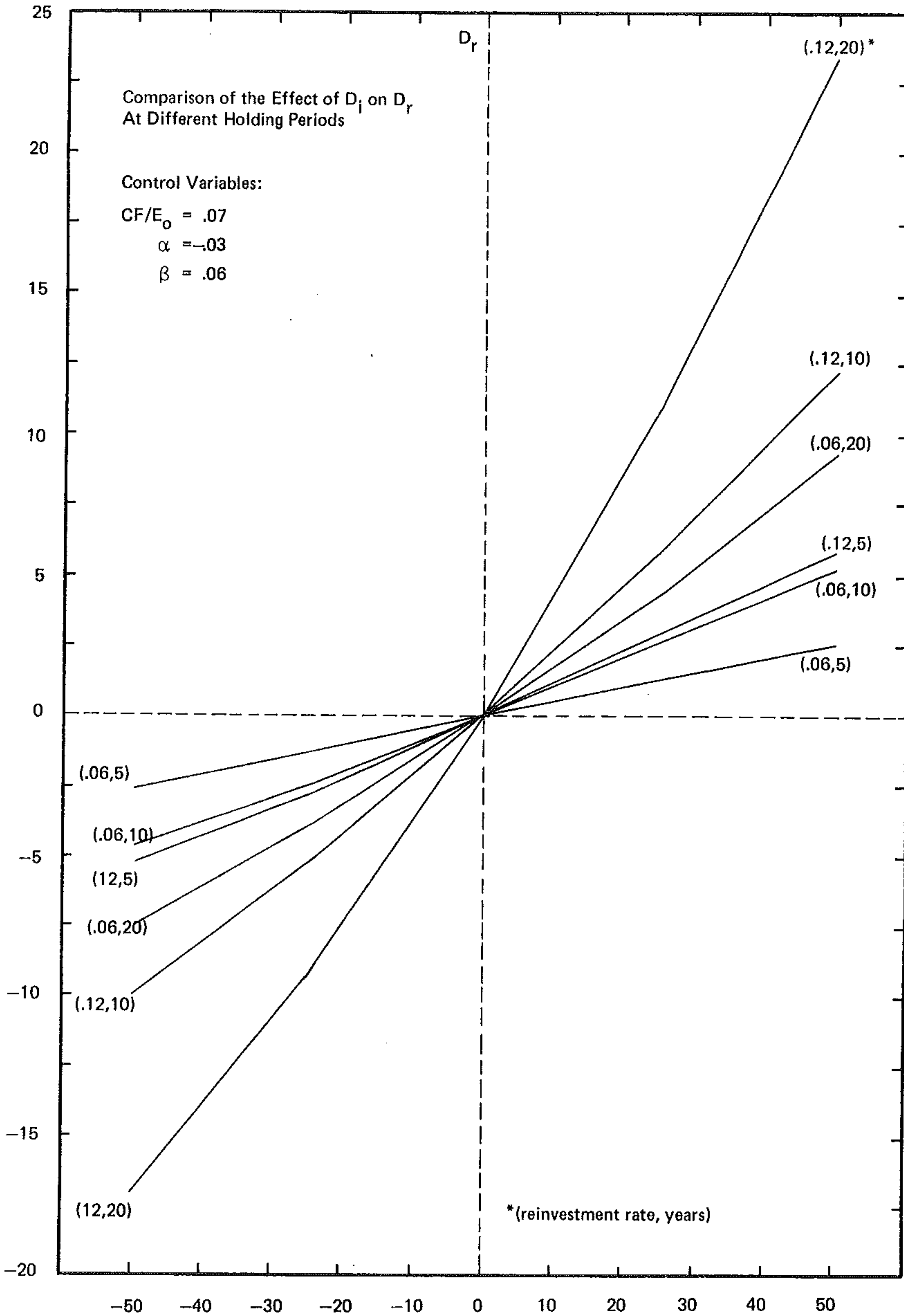
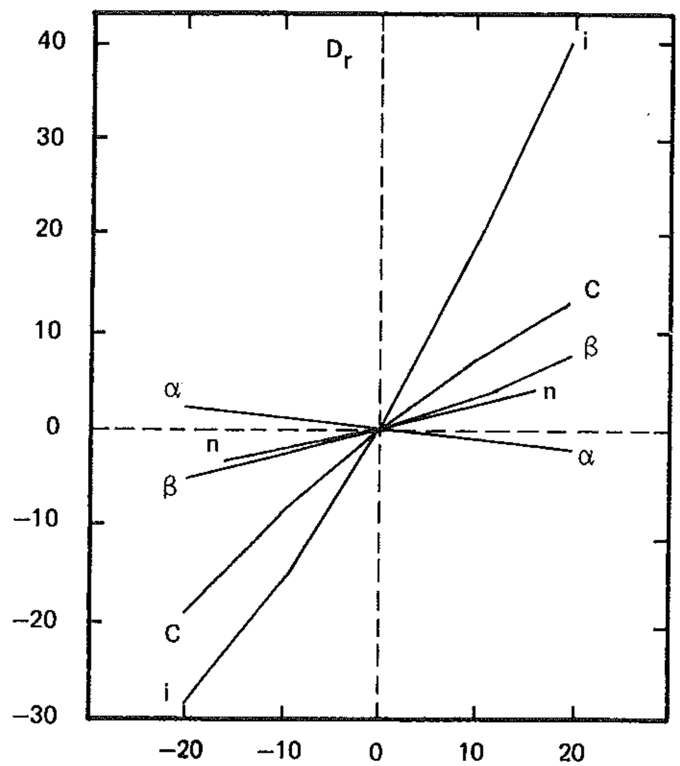

Comparison of the Effect of D_i on D_r
At Different Holding Periods
Control Variables:
CF/E_o = .07
α = -.03
β = .06
D_r
(.12,20)*
(.12,10)
(.06,20)
(.12,5)
(.06,10)
(.06,5)
(.06,5)
(.06,10)
(12,5)
(.06,20)
(.12,10)
(12,20)
*(reinvestment rate, years)
25
20
15
10
5
0
-5
-10
-15
-20
-50
-40
-30
-20
-10
0
10
20
30
40
50

FIGURE 7

CF/E_o = .07

α = .03
β = .06
i = .18
n = 20



FOOTNOTES

[1]Examples of the utilization of sensitivity analysis on a case study basis include Ratcliff and Schwab [9] and Cooper and Morrison [1].

[2]An alternative DCF framework would be the adaptation of this model to the determination of the rate of return on total capitalization. To separate the financing and investment decisions in capital budgeting, total capital is employed instead of E_o, and mortgage obligations are excluded from the calculation of both the net cash flows each period and the reversion at the end of the holding period. We choose to include the effect of financing in our development since the rate of return on equity seems to be the more widely applied measure in real estate investment analysis.

[3]While Wendt and Cerf [10, pp. 32-34] do mention the reinvestment rate in their analysis, they consider this rate in the role of a "Fisher rate of return over cost"—a rate of discount that equates the rates of return of two cash flow streams with alternative holding periods— rather than as an input in the calculation of a decision measure.

[4]For a description of the net terminal value technique, see Porterfield [8, pp. 38-41].

[5]Of course, situations can easily be envisioned where α and/or β assume the opposite signs. As an example, with a very high growth rate of the rents, little growth of the expenses and a straight-line depreciation method, the CF of an investment would increase during the holding period. Conversely, a deteriorating condition of the real estate market for the property may be such that R_n declines from $t = o$ to $t = n$.

[6]These values were derived from a case analysis by Cooper and Phyrr [2]. This case has been used in several other articles demonstrating discounted cash flow models, and most recently it has been used by Wendt and Tandy [11] in a paper contrasting different alogrithms used to calculate the rate of return in real estate investments. The authors employed these values as representations of an appropriate level of variable values for real estate investments.

[7]To maintain integer values for n^a, the D_n's were limited to positive and negative 20 percent and 40 percent.

[8]α must have a negative value for increases in the cash flow so that the term $(1 - \alpha)$ from equation (14) will be greater than one when the substitution is made.

[9]Because they were felt to be of more interest than the positive errors, only the results of the negative errors of estimation with $n = 5$ and $n = 20$ are detailed in Table 1. The values from the runs with positive errors and $n = 10$ can be approximated from an examination of Figure 1-5.

[10]$E_{r:1} = .149$ should be read, "the elasticity of r with respect to i equals .149."

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